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Epoch of Recombination (Cont'd):

As we saw, the majority of electrons got bound to protons at a redshift $z_{rec} \sim 1200 - 1400$. At the time $t_{rec} \sim 400,000$ yr, when $T_{rec} \sim 0.3$ eV, only about 10% of electrons were still free. However, in an expanding universe combination of electrons and protons drops out of equilibrium inevitably, thus leaving us with a tiny residual ionization fraction. As tiny as it may be ($X_\infty \sim 3\times10^{-3}$ as mentioned), it can still have an important impact on the photon-electron scattering, which eventually determines when the universe became transparent to photons. Here, we are going to have a more careful analysis of the important physical processes that are involved.

Recombination (Closer Look):

First consider the following reaction:

$e^- + p \rightleftarrows H + \gamma$

We need to find the rate at which e^- and combine and compare that to the Hubble expansion rate. The rate should be calculated for an electron with kinetic energy $\frac{1}{2} m_e v^2$ and then averaged over the thermal ensemble. Here we present an approximate method to estimate the rate for e^- and p combination.

First, we note that $\vec{v}_{rel}$ is essentially the velecity of the electron since proton has a much smaller velocity due to its larger mass. We consider temperatures $T \lesssim O(eV)$, recall that $T_{eq} \sim O(eV)$, and focus on the formation of Hydrogen atoms in the ground state.

The cross section for an electron with kinetic energy $E_{kin} \lesssim O(eV)$ to end up in the ground state of a neutral atom is $\sim \pi r_{rec}^2$ where $r_{rec} \sim \frac{e^2}{B}$. Here $B = 13.6$ eV is the ground state energy. This estimate assumes that the electron is localized within region with a size smaller than r_{rec}. However, there is an intrinsic quantum mechanical spread that is given by the de Broglie wavelength of the electron $\lambda_{deB} \sim \frac{1}{m_e v}$. Therefore:

$$\sigma_{rec} = \max\left[\pi r_{rec}^2 , \pi \lambda_{deB}^2\right]$$

This results in:

$$\langle \sigma_{rec} v_{rel} \rangle \sim \frac{4\pi^2 \alpha}{m_e^2} \frac{B}{(m_e T)^{1/2}}$$

Here we have used $v_{rel}^2 \sim \frac{T}{m_e}$. In a more precise calculation, one finds the overlap between the wave functions of a free electron with momentum $m_e v$ and the n-th bound state of the Hydrogen atom ($n=1$ being the ground state). This results in (after taking the thermal average):

$$\langle \sigma_{rec} v_{rel} \rangle = \sum_{n=1}^{\infty} \langle \sigma_n v_{rel} \rangle$$

$$\langle \sigma_n v_{rel} \rangle = \frac{4\pi^2 \alpha}{m_e^2} \frac{B}{n(3 m_e T)^{1/2}}$$

It is seen that the contribution from the ground state dominates, which leads to:

$$\langle \sigma_{rec} v_{rel} \rangle \sim \frac{4\pi^2 \alpha}{m_e^2} \frac{B}{3(m_e T)^{1/2}} = 4.7 \times 10^{-24} \left(\frac{1\,eV}{T}\right)^{1/2} cm^2 \quad (I)$$

We can then find $\Gamma_{rec} = n_p \langle \sigma_{rec} v_{rel} \rangle$, where $n_p = n_e = X_e n_B$

and X_e is taken from the Saha equation:

$$\frac{1 - X_e}{X_e^2} = \frac{4\sqrt{2}\,\zeta(3)}{\sqrt{\pi}} \eta \left(\frac{T}{m_e}\right)^{3/2} \exp\left(\frac{B}{T}\right)$$

For $X_e \ll 1$, which is the case for $t > t_{rec}$, we have:

$$X_e \simeq \left[\frac{4\sqrt{2}\,\zeta(3)}{\sqrt{\pi}} \eta \left(\frac{T}{m_e}\right)^{3/2} \exp\left(\frac{B}{T}\right)\right]^{-\frac{1}{2}}$$

It turns out that $\Gamma_{rec} \lesssim H$ at a redshift:

$$z_{f.o.} \sim 1080 - 1180 \quad (II)$$

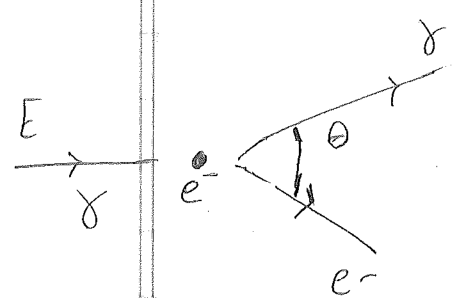
This is the time when electron and proton combination freezes out. This freeze out results in a residual ionization fraction $X_\infty \sim 3 \times 10^{-3}$.

Next, we consider photon-electron scattering:

$$e^- + \gamma \to e^- + \gamma$$

This is the Compton scattering, which plays the main role in keeping photons and electrons coupled to each other. At temperatures that we consider $T \lesssim O(eV)$ higher order processes like the double (inverse) Compton scattering $e^- + \gamma \to e^- + \gamma + \gamma$ are not significant.

The cross section for the Compton scattering can be calculated in quantum electrodynamics. Consider scattering of a photon with energy E off an electron that is at rest:



$$\frac{E'}{E} = \frac{1}{1 + \frac{E}{m_e}(1 - \cos\theta)}$$

It is seen that $E' \leq E$, and only for the forward scattering $\theta = 0$ we have $E' = E$. The partial cross section is given by the so-called Klein-Nishina formula:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi \alpha^2}{m_e^2} \left(\frac{E'}{E}\right)^2 \left[\frac{E'}{E} + \frac{E}{E'} - \sin^2\theta\right]$$

We note that for $E \ll m_e$ (as is the case at temperatures of interest for us) we have $E' \simeq E$. The Klein-Nishina formula is then reduced to:

$$\frac{d\sigma}{d\cos\theta} \simeq \frac{\pi \alpha^2}{m_e^2}(1+\cos^2\theta) \Rightarrow \sigma = \frac{8\pi\alpha^2}{3 m_e^2} \qquad (\text{III})$$

But this is the cross section for Thomson scattering σ_T, which we discussed last time and can be calculated in classical electrodynamics. Therefore, as expected, at low energies $E \ll m_e$ Compton scattering reproduces Thomson scattering.

The rate for photon-electron scattering is thus given by: (note that $v_{rel} = 1$)

$$\Gamma_T = n_e \sigma_T = X_e n_B \sigma_T \qquad (\text{IV})$$

After using $X_e = X_\infty \sim 3 \times 10^{-3}$, we find that $\Gamma_T \lesssim H$ at:

$$z_{dec} \sim 1100 - 1200 \qquad (\text{V})$$

This is slightly later than the freeze out of electron and proton combination, which justifies using X_∞ in Eq. (IV).

Decoupling of photons occurs at $t_{dec} \sim 500{,}000$ yr when $T_{dec} \simeq 0.26$ eV.

After recombination the pressure provided by photons (through scattering off electrons) significantly decreases. As a result, baryons will not feel any significant pressure and can freely fall in the potential wells from density fluctuations. After decoupling of photons, they can freely move without further scattering off the electrons. These are the CMB photons that we observe now and provide a snapshot of the universe at t_{dec}.